2.4 - Basic Trigonometric Identities

Warmup - Evaluate if $x = \pi$.

1. $5 + \frac{3}{2}\sin x$ $\quad 5$

2. $2\cos\frac{x}{4}$ $\quad \sqrt{2}$

3. $3\sin\frac{x}{6} - \frac{1}{2}$ $\quad 1$

4. $3 + 5\cos 3x$ $\quad -2$

Basic Trig Identities

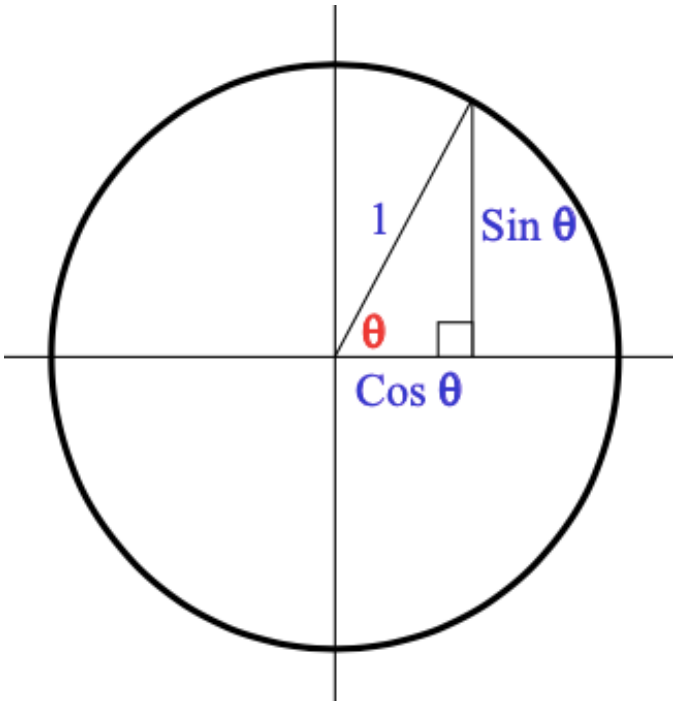
2.4 - Basic Trigonometric Identities

What's the relationship between $\sin\theta$, $\cos\theta$, and 1?

$$\sin^2\theta + \cos^2\theta = 1^2$$

or

$$\sin^2\theta + \cos^2\theta = 1$$



2.4 - Basic Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

Divide by $\sin^2\theta$?

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Divide by $\cos^2\theta$?

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Sample Problems

$$\frac{\cos\theta}{\sec\theta} = \frac{\cos\theta}{\frac{1}{\cos\theta}} = \cos^2\theta$$

$$\frac{1-\cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

2.4 - Basic Trigonometric Identities

Fundamental Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \csc^2\theta$ $\quad \tan^2\theta + 1 = \sec^2\theta$

$\csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

Practice

1. $\dfrac{1}{\cot^2\theta} - \dfrac{1}{\cos^2\theta}$ $\quad -1$

2. $\dfrac{1 + \cot^2\theta}{\sec^2\theta}$ $\quad \cot^2\theta$

3. $\tan^2\theta + \csc^2\theta - \cot^2\theta$ $\quad \sec^2\theta$

2.4 - Basic Trigonometric Identities

Fundamental Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \csc^2\theta$ $\quad \tan^2\theta + 1 = \sec^2\theta$

$\csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

Practice

1. $\sin^2\theta(1 + \cot^2\theta)$ $\quad$ 2. $\dfrac{\tan\theta + \cot\theta}{\sec^2\theta}$ $\quad$ 3. $\dfrac{\sec\theta - \cos\theta}{\tan^2\theta}$

1 $\quad \cot\theta$ $\quad \cos\theta$

2.4 - Basic Trigonometric Identities

Warmup -

1. $\cos 2670^\circ$ $\quad -\frac{\sqrt{3}}{2}$

2. $\cot(-1575^\circ)$ $\quad 1$

3. $\csc\left(\frac{107\pi}{6}\right)$ $\quad -2$

2.4 - Basic Trigonometric Identities

Fundamental Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \csc^2\theta$ $\quad \tan^2\theta + 1 = \sec^2\theta$

$\csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

Practice - Reduce

1. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

$2\csc\theta$

2. $\dfrac{\tan\theta + \cot\theta}{\sec\theta\csc\theta}$

1

2.4 - Basic Trigonometric Identities

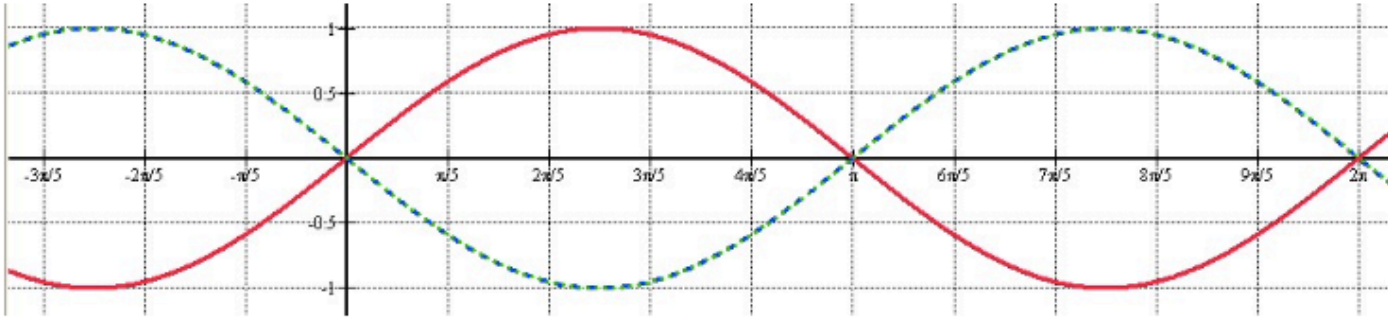
Odd Identities $f(-x) = -f(x)$

$\sin(-x) = -\sin(x)$

$\csc(-x) = -\csc(x)$

$\tan(-x) = -\tan(x)$

$\cot(-x) = -\cot(x)$

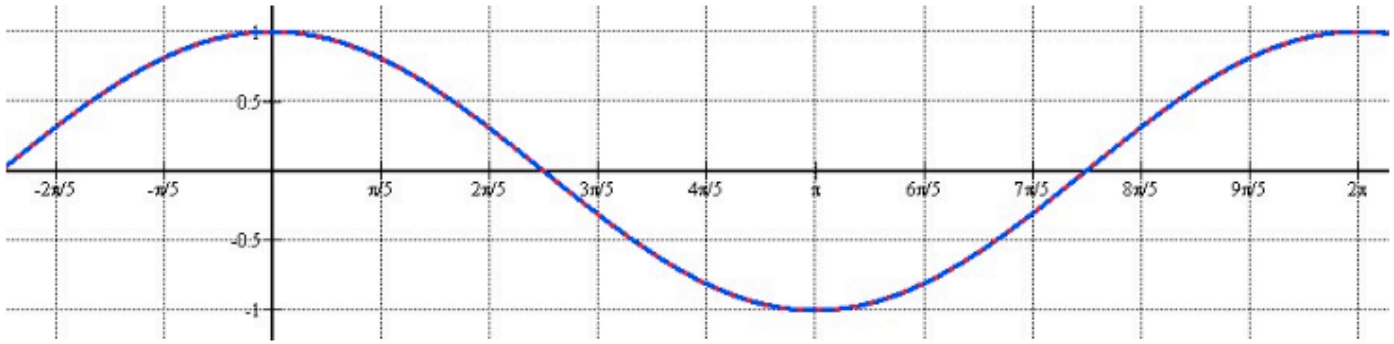


Even Identities

$f(-x) = f(x)$

$\cos(-x) = \cos(x)$

$\sec(-x) = \sec(x)$



What about $\sin^2(-x)$? $\sin^2(-x) = \sin^2(x)$

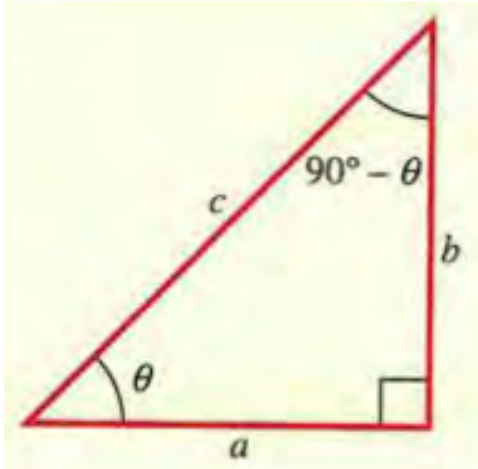
1.3 - Right Triangle Ratios

Cofunction identities

$\sin\theta = \cos(90^\circ - \theta)$ $\quad$ $\cos\theta = \sin(90^\circ - \theta)$

$\tan\theta = \cot(90^\circ - \theta)$ $\quad$ $\cot\theta = \tan(90^\circ - \theta)$

$\sec\theta = \csc(90^\circ - \theta)$ $\quad$ $\csc\theta = \sec(90^\circ - \theta)$



Practice

1. $\cos(90 - x) \cdot \sec(-x) = \tan(x)$

2. $\tan(90 - x) \cdot \csc(90 - x) \cdot \sin(-x) = -1$

3. $\dfrac{\cos x}{1 + \sin(-x)} + \dfrac{1 - \sin x}{\sin(90 - x)} = 2\sec x$

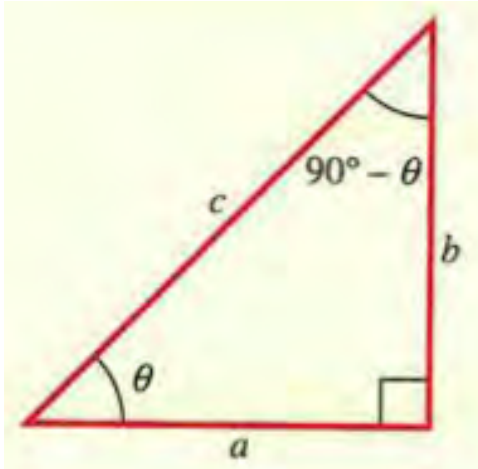
1.3 - Right Triangle Ratios

Cofunction identities

$\sin\theta = \cos(90^\circ - \theta)$ $\quad\quad$ $\cos\theta = \sin(90^\circ - \theta)$

$\tan\theta = \cot(90^\circ - \theta)$ $\quad\quad$ $\cot\theta = \tan(90^\circ - \theta)$

$\sec\theta = \csc(90^\circ - \theta)$ $\quad\quad$ $\csc\theta = \sec(90^\circ - \theta)$



Practice

1. $\tan^2(x) - 4\tan(x) - 5 = 0$

78.7°, 315°, 258.7°, 135°

2. $\sec^2(x) - 8\sec(-x) - 20 = 0$

84.3°, 275.7°, 120°, 240°

2.4 - Basic Trigonometric Identities

Solving Trig Equations

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \csc^2\theta$ $\quad \tan^2\theta + 1 = \sec^2\theta$

$2(1 - \cos^2 x) + \sin x - 1 = 0$ for $0° \le x < 360°$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}, \ \sin x = -1$

$x = 30°, 150°, 270°$

2.4 - Basic Trigonometric Identities

Solving Trig Equations

$2\sin^2 x - 1 = 0 \quad \text{for } 0° \le x < 360°$

$2\sin^2 x = 1 \qquad \sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2} \qquad x = 45°, 135°, 225°, 315°$

Practice

1. $\sin^2(x) - \sin(x) = \cos^2(x)$

$x = 90°, 210°, 330°$

2. $\sin(x) \cdot \tan(x) = 3\sin(x)$

$x = 0°, 71.6°, 180°, 251.6°$

2.4 - Basic Trigonometric Identities

Solving Trig Equations

$2\sin^2 x - 1 = 0 \quad \text{for } 0° \le x < 360°$

$2\sin^2 x = 1 \qquad \sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2} \qquad x = 45°, 135°, 225°, 315°$

Practice

1. $\tan^2 x + 2\tan x - 22 = 0$

$\tan x = 3.8, -5.8$

$x = 75.2°, 99.8°, 279.8°, 255.2°$

2. $\cot^4 x - 3\cot^2 x + 2 = 0$

$x = 35.3°, 144.7°, 215.3°, 324.7°$

$45°, 135°, 225°, 315°$